After your PhD, there are four main career moves you can make.

1. You can leave academia. People will be impressed if you have a PhD from a very big name university. It won’t count for much compared to just the prestige of having a PhD, which also doesn’t count for much. The skills you have learnt during your PhD might matter a lot, and might be very marketable if you play your cards right, but that has nothing to do with where you got the PhD.
2. You can go into a contract postdoc position. These positions are often not publicly advertised, and so most of the challenge is finding the positions, rather than winning a competitive recruitment. Finding the position depends on your own network and the network of your supervisors. The prestige of your supervisors/advisors/examining committee will matter rather than the university itself for this. It’s really hard for PhD candidates to know accurately how well connected their supervisors are. There are plenty of academics in 301–350 universities who have networks that include top 10 institutions.
3. You can go into a competitive postdoc fellowship position. These are publicly advertised, so there is usually a lot of competition. This will depend mainly on “objective” measures such as what you have published and where. It will depend on the strength of your project proposal. It will probably be swayed by where you did your PhD.
4. You can go straight into an academic post at a lower-ranked university. This may be swayed by where you did your PhD, but teaching experience/skill and publication record will matter a lot more. The better the university you are applying to, the less it will matter where you did your PhD and the more your own personal record will matter.